USN											10MAT31

Third Semester B.E. Degree Examination, June 2012
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions choosing atleast two from each part.***

PART – A

1 a. Obtain the Fourier series for the function

$$f(x)=\begin{cases}1+\dfrac{2x}{\pi}, & -\pi\le x\le 0\\ 1-\dfrac{2x}{\pi}, & 0\le x\le\pi\end{cases}$$ and deduce $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+........=\dfrac{\pi^2}{8}$. **(07 Marks)**

b. Find the half range cosine series for the function $f(x) = (x-1)^2$ in $0 < x < 1$ **(06 Marks)**

c. Obtain the constant term and the coefficient of the first sine and cosine terms in the Fourier expansion of y as given below. **(07 Marks)**

x	0	1	2	3	4	5
y	9	18	24	28	26	20

2 a. Express the function

$$f(x)=\begin{cases}1, & |x|\le 1\\ 0, & |x|>1\end{cases}$$ as a Fourier integral and hence evaluate $\displaystyle\int_0^\infty \frac{\sin x}{x}\,dx$. **(07 Marks)**

b. Find the sine and cosine transform of $f(x) = e^{-ax}$, $a > 0$ **(06 Marks)**

c. Find the inverse Fourier sine transform of $\dfrac{e^{-as}}{s}$. **(07 Marks)**

3 a. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is vibrating giving to each of its points a velocity $\lambda x(l - x)$, find the displacement of the string at any distance x from one end and at any time t. **(07 Marks)**

b. Find the temperature in a thin metal bar of length 1 where both the ends ate insulated and the initial temperature in bar is $\sin\pi x$. **(07 Marks)**

c. Find the solution of Laplace equation, $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$, by the method of separation of variables. **(06 Marks)**

4 a. Fit a parabola $y = a + bx + cx^2$ to the following data: **(07 Marks)**

x	-3	-2	-1	0	1	2	3
y	4.63	2.11	0.67	0.09	0.63	2.15	4.58

b. A fertilizer company produces two products Naphtha and Urea. The company gets a profit of Rs.50 per unit product of naphtha and Rs.60 per unit product of urea. The time requirements for each product and total time available in each plant are as follows:

Plant	Hours required		Available hours
	Naphtha	Urea	
A	2	3	1500
B	3	2	1500

The demand for product is limited to 400 units. Formulate the LPP and solve it graphically. **(06 Marks)**

c. Solve the following using Simplex method:
Maximize $Z = x_1 + 4x_2$
Subject to constraints $-x_1 + 2x_2 \le 6$; $5x_1 + 4x_2 \le 40$; $x_j \ge 0$. **(07 Marks)**

PART – B

5 a. Use Regula-falsi method to find a root of the equation $2x - \log_{10}x = 7$ which lies between 3.5 and 4. **(06 Marks)**

b. Solve by relaxation method.
$10x - 2y - 2z = 6$; $-x + 10y - 2z = 7$; $-x - y + 10z = 8$ **(07 Marks)**

c. Use the power method to find the dominant eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ with the initial eigenvector as $[1\ \ 1\ \ 1]^T$. **(07 Marks)**

6 a. The following data is on melting point of an alloy of lead and zinc where t is the temperature in Celsius and P is the percentage of lead in the alloy, tabulated for P = 40(10)90 (i.e., P from 40 to 90 at intervals of 10). Find the melting point of the alloy containing 86% of lead.

P	40	50	60	70	80	90
t	180	204	226	250	276	304

(07 Marks)

b. Using Lagrange's formula, find the interpolation polynomial that approximates to the functions described by the following table:

x	0	1	2	5
f(x)	2	3	12	147

and hence find f(3). **(07 Marks)**

c. Evaluate $\int_0^5 \frac{dx}{4x+5}$, by using Simpson's $\frac{1}{3}^{rd}$ rule, taking 10 equal parts. Hence find $\log 5$. **(06 Marks)**

7 a. Solve the partial differential equation
$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = -10\left(x^2 + y^2 + 10\right)$$
over the square with side x = 0, y = 0, x = 3, y = 3 with u_0 on the boundary and mesh length h = 1. **(07 Marks)**

b. Solve the heat equation $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}$, subject to the conditions
$U(0, t) = u(1, t) = 0$ and $u(x,0) = \begin{cases} 2x & \text{for } 0 \le x \le 1/2 \\ 2(1-x) & \text{for } 1/2 \le x \le 1 \end{cases}$
Taking h = 1/4 and according to Bender Schmidt equation. **(06 Marks)**

c. Evaluate the pivotal values of the equation $u_{tt} = 16\ u_{xx}$ taking h = 1 upto t = 1.25. The boundary conditions are $u(0, t) = u(5, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x^2(5 - x)$. **(07 Marks)**

8 a. If $U(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$, evaluate u_2 and u_3. **(06 Marks)**

b. Find the Z-transform of i) $\sin(3n + 5)$ ii) $\frac{1}{(n+1)!}$. **(07 Marks)**

c. Solve the $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-transforms. **(07 Marks)**

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